Algebra 1

Pacing Guide and Unpacked Standards

Developed by:
Gabrielle Kisner, GMLSD Teacher
Carri Meek, School Improvement Specialist,
Instructional Growth Seminars and Support
Garilee Ogden, GMLSD Director of Curriculum, Instruction and Professional Development

Resources: School District U-46, of Chicago, IL, The Ohio Department of Education,
Columbus City Schools, Common Core Institute and North Carolina Department of Public Instruction.

We would like to thank the teachers of GMLSD that provided feedback and support.

Groveport Algebra I Pacing Overview

➢ **Indicates Blueprint Focus Standards**

*** Indicates Modeling Standard**

Algebra I	Number, Quantities, Equations and Expressions	Functions	Statistics	Standards for Mathematical Practice
1st 9 Wks	➢**N.Q.1*** Choose and interpret units consistently in formulas ➢**N.Q.2*** Define appropriate quantities for descriptive modeling ➢**N.Q.3*** Choose appropriate limitations on measurements ➢**A.SSE.1(a, b)*** Interpret parts of an expression ➢**A.SSE.2** Identify ways to rewrite an expression ➢**A.CED.1(a, b)*** Create equations in one variable ➢**A.CED.3*** Represent constraints by equations ➢**A.CED.4(a, b)*** Rearrange formulas to highlight a quantity of interest ➢**A.REI.1** Explain each step in solving a simple equation ➢**A.REI.3** Solve linear equations in one variable	➢**A.REI.10** Understand that the graph of an equation is the set of all its solutions plotted on the coordinate plane ➢**F.IF.1** Understand properties of domain and range ➢**F.IF.2** Notate, evaluate and interpret functions ➢**F.IF.4*** Interpret key features of graphs/tables ➢**F.IF.5(a, b, c)*** Relate the domain of a function to its graph ➢**F.IF.7(a)*** Graph linear functions ➢**F.BF.2*** Write arithmetic and geometric sequences ➢**F.LE.1(a, b)*** Distinguish between linear and exponential functions ➢**F.LE.2*** Construct linear functions		**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision **MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning
2nd 9 Wks	➢**A.CED.1*** Create inequalities in one variable ➢**A.CED.2(a, b)*** Create equations and inequalities in two or more variables ➢**A.CED.3*** Represent constraints by inequalities ➢**A.REI.3** Solve linear inequalities in one variable	➢**A.REI.5** Verify that replacing one equation in a system by its sum and the multiple of the other produces the same solutions ➢**A.REI.6** Solve systems of linear equations **A.REI.11** Explain why the x-coordinates of intersecting graphs are solutions **A.REI.12** Graph the solutions to linear inequalities using half planes ➢**F.IF.2** Notate, evaluate and interpret functions ➢**F.IF.6*** Calculate average rate of change ➢**F.IF.7(a)*** Graph linear functions ➢**F.IF.9(a)** Compare properties of two functions ➢**F.BF.1(a, b)*** Write a function that describes a relationship between two quantities **F.BF.4(a)** Find inverse functions ➢**F.LE.2*** Construct	➢**S.ID.6(a, b, c)*** Represent data in two variables on a scatter plot ➢**S.ID.7*** Interpret the slope and the intercept of a linear model ➢**S.ID.8*** Compute and interpret the correlation coefficient of a linear fit	

Groveport Algebra I Pacing Overview

➢ Indicates Blueprint Focus Standards
* Indicates Modeling Standard

		linear/exponential functions		
Algebra I	**Number, Quantities, Equations and Expressions**	**Functions**	**Statistics**	**Standards for Mathematical Practice**
3rd 9 Wks	➢**A.SSE.2** Identify ways to rewrite an expression ➢**A.SSE.3(a, b, c)*** Factor, complete the square and use properties of exponents to transform quadratic expressions ➢**A.APR.1** Add, subtract and multiply polynomials ➢**A.REI.4(a)** Solve quadratic equations **A.REI.7** Solve a simple system consisting of a linear and a quadratic equation ➢**A.CED*** Represent constraints by systems of equations	➢**N.RN.1** Explain the meaning of rational exponents ➢**F.IF.3** Recognize that sequences are functions ➢**F.IF.4(a, b)*** Interpret and sketch key features of graphs **F.BF.4(a)** Find inverse functions ➢**F.IF.7(a, b, e)*** Indicate key features of a graph ➢**F.IF.8(a, b)** Write functions in equivalent forms ➢**F.IF.9(a, b)** Compare properties of two functions ➢**F.BF.2*** Write arithmetic and geometric sequences ➢**F.BF.3(a)** Transform graphs of quadratic functions ➢**F.LE.1(a, b, c)*** Distinguish between linear and exponential functions ➢**F.LE.2*** Construct linear and exponential functions ➢**F.LE.3*** Observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly/quadratically ➢**F.LE.5*** Interpret the parameters in a linear or exponential function		**MP.1** Make sense of problems and persevere in solving them **MP.2** Reason abstractly and quantitatively **MP.3** Construct viable arguments and critique the reasoning of others **MP.4** Model with mathematics **MP.5** Use appropriate tools strategically **MP.6** Attend to precision **MP.7** Look for and make use of structure **MP.8** Look for and express regularity in repeated reasoning
4th 9 Wks	➢**A.CED.2*** Create and graph rational and radical equations **A.REI.2** Solve rational and radical equations in one variable ➢**A.REI.4(b)** Solve quadratic equations	➢**N.RN.2** Rewrite radical and rational expressions using exponent properties ➢**N.RN.3** Explain properties of operations with rational and irrational numbers ➢**F.IF.4*** Interpret and sketch key features of graphs **F.IF.7(b)*** Graph and indicate key features of quadratic functions	➢**S.ID.1*** Represent data with plots on the number line ➢**S.ID.2*** Use statistics to compare center and spread of two or more data sets ➢**S.ID.3*** Interpret differences in shape, center and spread in the context of data sets ➢**S.ID.5*** Summarize categorical data in two-way frequency tables ➢**S.ID.9*** Distinguish between correlation and causation	

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

N.RN.1-3

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational 1/3 exponents. For example, we define 5^1/3 to be the cube root of 5 because we want (5^1/3)3 = 5^(1/3)3 to hold, so (5^1/3)3 must equal 5.

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

3. Explain why sums and products of rational numbers are rational,that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

Essential Understanding

Students use laws of exponents to understand radicals as rational exponents.

Extended Understanding

Students use logarithms to solve exponential equations.

Vocabulary

- Define
- Denominator
- Explain
- "In terms of"
- Numerator
- Properties of exponents
- Radicand
- Rational

Essential Skills

- I can define radical notation as a convention used to represent rational exponents.
- I can explain the properties of operations of rational exponents as an extension of the properties of integer exponents.
- I can explain how radical notation, rational exponents, and properties of integer exponents relate to one another.
- I can, using the properties of exponents, rewrite a radical expression as an expression with a rational exponent.
- I can, using the properties of exponents, rewrite an expression with rational exponent as a radical expression.

Instructional Strategies The goal is to show that a fractional exponent can be expressed as a radical or a root. For example, an exponent of 1/3 is equivalent to a cube root; an exponent of ¼ is equivalent to a fourth root. Review the power rule, $((b^n)^m = b^{mn})$, for whole number exponents (e.g. $(7^2)^3 = 7^6$. Compare examples, such as $\left(7^{\frac{1}{2}}\right)^2 = 7^1 = 7$ and $\left(\sqrt{7}\right)^2 = 7$, to help students establish a connection between radicals and rational exponents: $7^{\frac{1}{2}} = \sqrt{7}$ and, in general, $b^{\frac{1}{2}} = \sqrt{b}$. Provide opportunities for students to explore the equality of the values using calculators, such as $7^{\frac{1}{2}}$and $\sqrt{7}$. Offer sufficient examples and exercises to prompt the definition of fractional exponents, and give students practice in converting expressions between radical and exponential forms. When n is a positive integer, generalize the meaning of $b^{1/n} = \sqrt[n]{b^1}$ and then to $b^{m/n} = \sqrt[n]{b^m}$, where n and m are integers and n is greater than or equal to 2. When m is a negative integer, the result is the reciprocal of the root $b^{-m/n} = \frac{1}{\sqrt[n]{b^m}}$. Stress the two rules of rational exponents: 1) the numerator of the exponent is the base's power, and 2) the denominator of the exponent is the order of the root. When evaluating expressions involving rational exponents, it is often helpful to break an exponent into its parts – a power and a root – and then decide if it is easier to perform the root operation or the exponential operation first.
Common Misconceptions and Challenges Students sometimes misunderstand the meaning of exponential operations, the way powers and roots relate to one another, and the order in which they should be performed. Attention to the base is very important. Consider examples: $(-81^{\frac{3}{4}})$ and $(-81)^{3/4}$. The position of a negative sign of a term with a rational exponent can mean that the rational exponent should be either applied first to the base, 81, and then the opposite of the result is taken $(-81^{\frac{3}{4}})$, or the rational exponent should be applied to a negative term $(-81)^{3/4}$. The answer of $\sqrt[4]{-81}$ will be not real if the denominator of the exponent is even. If the root is odd, the answer will be a negative number. Students should be able to make use of estimation when incorrectly using multiplication instead of exponentiation. Students may believe that the fractional exponent in the expression $36^{\frac{1}{3}}$ means the same as a factor $\frac{1}{3}$ in multiplication expression, $36 \cdot \frac{1}{3}$ and multiply the base by the exponent.
Performance Level Descriptors **Limited:** N/A **Basic:** N/A **Proficient:** N/A **Accelerated:** N/A **Advanced:** Accurately multiply polynomials of any number of terms using rules of exponents

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

N.Q.1, N.Q.2, N.Q.3

Reason quantitatively and use units to solve problems.

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays; define appropriate quantities for the purpose of descriptive modeling; choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Essential Understanding

Students must be able to reason abstractly and quantitatively, making sense of quantities and their relationships in problem situations. This involves the ability to create coherent representation(s) of the problem at hand, considering the units involved, attending to the meaning of quantities, and knowing and flexibly using different properties of operations and objects.

Extended Understanding

Students will continue using quantities to model and analyze situations, interpret expressions, and create equations to describe situations.

Vocabulary

- Acceleration
- Approximation
- Area
- Average
- Conversion
- Data
- Length
- Limitations
- Proportion
- Quantity(ies)
- Rates
- Ratio
- Scale
- Unit of measure
- Variables
- Volume

Essential Skills

- I can use units to evaluate the appropriateness of the solutions when solving multi- step problems.
- I can choose the appropriate units for a specific formula and interpret the meaning of the unit in that context.
- I can choose and interpret both the scale and the origin in graphs and data displays.
- I can determine and interpret appropriate quantities when using descriptive modeling.
- I can determine the accuracy of values based on their limitations in the context of the situation.

Instructional Strategies

At the high school level, math dealing with number and quantity expands to include exponents as well as imaginary and complex numbers. Students experience a broader variety of units through real - world situations and modeling along with the exploration of the different levels of accuracy and precision of answers. Teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, etc.

Common Misconceptions and Challenges

Students struggle with units and unit conversions. Students may not realize the importance of the units' conversions in conjunction with the computation when solving problems involving measurements.

Since today's calculating devices often display 8 to 10 decimal places, students frequently express answers to a much greater degree of precision than the required.
Students must be able to identify and apply appropriate units and quantities when modeling real-world situations and reasoning through possible solutions within context; therefore, units should be chosen carefully to achieve a suitable level of accuracy and precision in the context of the problem.

Students should be able to recognize the relationship between the contextual, graphical, numeric and symbolic representations of a problem situation. It may be difficult for students to conceptualize that an equation, a table of values, and a graph can all represent the same problem situation.

Performance Level Descriptors

Limited: Identify units in familiar formulas involving whole numbers

Basic: Choose and interpret units in formulas; giving a situation, context, or problem, students will identify and use appropriate quantities for representing the situation

Proficient: Choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays

Accelerated: N/A

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.SSE.1-2

1. Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

2. Use the structure of an expression to identify ways to rewrite it. For example, to factor 3x(x – 5) +2(x – 5), students should recognize that the "x – 5" is common to botth expressions being added, so it simplifies to (3x + 2)(x – 5)

<u>Essential Understanding</u>

- Students will become fluent in identifying parts of an expression.

- Students will understand what is meant by *coefficient*, *factor*, and *term*.

<u>Extended Understanding</u>

- Students will be able to examine a large expression and see parts of it as single entities.

<u>Vocabulary</u>

- Coefficient
- Context
- Expression
- Factor
- Identify
- Interpret
- Represent
- Single entity
- Structure
- Term
- Use
- Viewing

Essential Skills

- I can, for expressions that represent a contextual quantity, define and recognize parts of an expression, such as terms, factors, and coefficients.
- I can, for expressions that represent a contextual quantity, interpret parts of an expression, such as terms, factors, and coefficients in terms of the context.
- I can, for expressions that represent a contextual quantity, interpret complicated expressions, in terms of the context, by viewing one or more of their parts as a single entity.

Instructional Strategies

Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “2x” and “1” being *terms* of the binomial expression. Development and proper use of mathematical language is an important building block for future content.

Using real-world context examples, the nature of algebraic expressions can be explored. For example, suppose the cost of cell phone service for a month is represented by the expression $0.40s + 12.95$. Students can analyze how the coefficient of 0.40 represents the cost of one minute (40 cents), while the constant of 12.95 represents a fixed, monthly fee, and s stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.

Factoring by grouping is another example of how students might analyze the structure of an expression. To factor $3x(x - 5) + 2(x - 5)$, students should recognize that “$x - 5$” is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$. Students should become comfortable with rewriting expressions in a variety of ways until a structure emerges.

Have students create their own expressions that meet specific criteria (e.g., number of terms factorable, difference of two squares, etc.) and verbalize how they can be written and rewritten in different forms. Additionally, pair/group students to share their expressions and rewrite one another’s expressions.

Common Misconceptions and Challenges

Students may believe that use of algebraic expressions is merely the abstract manipulation of symbols. Use of real-world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.

Students may also believe that an expression cannot be factored because it does fit into a form they recognize. They need help with reorganizing the terms until structures become evident.

Performance Level Descriptors

Limited: Identify parts of simple linear expressions, terms, factor and coefficient

Basic: Identify parts of simple linear expressions in terms of the context the quantity represents: terms, factors and coefficients

Proficient: Interpret parts of a simple exponential expression in terms of its context; recognize the structure of a quadratic expression to identify ways to rewrite it to better represent the purpose

Accelerated: Interpret linear expressions by viewing one or more of their parts as a single entity in respect to the context; use the structure of an exponential expression to identify ways to rewrite it

Advanced: Interpret exponential expressions by viewing one or more of their parts as a single entity in relationship to the context

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.SSE.3ab

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

<u>Essential Understanding</u>

- Students will know how to factor.

- After factoring, students will set each factor to zero and find the value of each root.

<u>Extended Understanding</u>

- Students will learn how to solve a quadratic equation by completing the square.

<u>Vocabulary</u>

- Choose
- Explain
- Expression
- Factor
- Maximum/minimum
- Produce
- Properties
- Quadratic expression
- Quantity
- Represented
- Reveal
- Use

Essential Skills

- I can factor a quadratic expression to produce an equivalent form of the original expression.
- I can explain the connection between the factored form of a quadratic expression and the zeros of the function it defines.
- I can explain the properties of the quantity represented by the quadratic expression.
- I can choose and produce an equivalent form of a quadratic expression to reveal and explain properties of the quantity represented by the original expression.

Instructional Strategies

When teaching completing the square, it can be helpful to start with problems where the value of **a** is 1. Also, the value of **b** should be an even number.

Common Misconceptions and Challenges

Remind students that completing the square requires the value of **a** to be 1. They may have to divide first.

Students often have difficulty when working with fractions. For instance, if the value of **b** is odd, dividing by 2 will result in a fraction. Students also will need to know how to square a fraction.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: Factor a quadratic expression to reveal the zeros of the function it defines

Accelerated: N/A

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.SSE.1ab, A.SSE.3c

Interpret structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.

b. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

3c. Use the properties of exponents to transform expressions for exponential functions. For example, 8^t can be written as 2^{3t}.

Essential Understanding

Students, given an expression, are expected to be able to evaluate it correctly for various values of the variables; given an expression or expressions with variables defined in an application, students are expected be able to evaluate/explain the expressions and be able to interpret the expressions based on the operations and uses of the variables.

Students are expected to know that changing the forms of expressions, such as factoring or completing the square, or transforming expressions from one exponential form to another, are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).

Extended Understanding

Students can research the natural number, *e*, and interpret its context in an exponential expression.

Vocabulary

- Arithmetic
- Coefficient
- Constant
- Geometric sequence
- Independent quantities
- Interpret
- Linear
- Numerical expression
- Sequence

Essential Skills

- I can identify the different parts of the expression and explain their meaning within the context of a problem.
- I can write expressions in equivalent forms by factoring to find the zeros of a quadratics function and explain the meaning of zeros.
- I can use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.

Instructional Strategies

Check student understanding of simplifying expressions with exponents. Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression $2x + 1$, "2" is the coefficient, "2" and "x" are factors, and "1" is a constant, as well as "$2x$" and "1" being *terms* of the binomial expression. Development and proper use of mathematical language is an important building block for future content.

Using real-world context examples, the nature of algebraic expressions can be explored. For example, suppose the cost of cell phone service for a month is represented by the expression $0.40s + 12.95$. Students can analyze how the coefficient of 0.40 represents the cost of one minute (40¢), while the constant of 12.95 represents a fixed, monthly fee, and s stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.

Factoring by grouping is another example of how students might analyze the structure of an expression. To factor $3x(x – 5) + 2(x – 5)$, students should recognize that the "$x – 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x – 5)$. Students should become comfortable with rewriting expressions in a variety of ways until a structure emerges.

Common Misconceptions and Challenges

Students often do not make the connections between expressions and their context. Specifically, the connection between the individual terms of the expression and the different aspects of the context.

Students who still struggle with integer and rational operation naturally have great difficulties when interpreting and creating expressions. Students often confuse the terms *equation*, *function*, and *expression* and use them interchangeably.

Performance Level Descriptors

Limited: Identify parts of simple linear expressions, terms, factor and coefficient

Basic: Identify parts of simple linear expressions in terms of the context the quantity represents: terms, factors and coefficients

Proficient: Interpret parts of a simple exponential expression in terms of its context

Accelerated: Interpret linear expressions by viewing one or more of their parts as a single entity in respect to the context

Advanced: Interpret exponential expressions by viewing one or more of their parts as a single entity in relationship to the context

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.APR.1a	Essential Understanding	Vocabulary
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms that are linear or quadratic.	- Students can add, subtract, and multiply polynomials. **Extended Understanding** - Students can divide polynomials. - Students can factor polynomials.	- Analogous - Binomial - Distribute - Like terms - Monomial - Polynomial - Trinomial - Understand

Essential Skills

- I can identify that the sum, difference, or product of two polynomials will always be a polynomial, which means that polynomials are closed under the operations of addition, subtraction, and multiplication.
- I can define "closure".
- I can apply arithmetic operations of addition, subtraction, and multiplication to polynomials.

Instructional Strategies

The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.

In arithmetic of polynomials, a central idea is the distributive property, because it is fundamental not only in polynomial multiplication but also in addition and subtraction. With the distributive property, there is little need to emphasize misleading mnemonics, such as FOIL, which is relevant only when multiplying two binomials, and the procedural reminder to "collect like terms" as a consequence of the distributive property. For example, when adding the polynomials 3x and 2x, the result can be explained with the distributive property as follows: $3x + 2x = (3 + 2)x = 5x$.

The new idea in this standard is called *closure*: A set is *closed* under an operation if when any two elements are combined with that operation, the result is always another element of the same set. In order to understand that polynomials are closed under addition, subtraction and multiplication, students can compare these ideas with the analogous claims for integers: The sum, difference or product of any two integers is an integer, but the quotient of two integers is not always an integer.

Common Misconceptions and Challenges

Some students will apply the distributive property inappropriately. Emphasize that it is the *distributive property of multiplication over addition*. For example, the distributive property can be used to rewrite $2(x + y)$ as $2x + 2y$, because in this product the second factor is a sum (i.e., involving addition). But in the product $2(xy)$, the second factor, (xy), is itself a product, not a sum.
Some students will still struggle with the arithmetic of negative numbers. Consider the expression $(-3) \cdot (2 + (-2))$. On the one hand, $(-3) \cdot \big(2 + (-2)\big) = (-3) \cdot (0) = 0$. But using the distributive property, $(-3) \cdot \big(2 + (-2)\big) = (-3) \cdot (2) + (-3) \cdot (-2)$. Because the first calculation gave 0, the two terms on the right in the second calculation must be opposite in sign. Thus, if we agree that $(-3) \cdot (2) = -6$, then it must follow that $(-3) \cdot (-2) = 6$.

Performance Level Descriptors

Limited: Use algebra manipulatives or diagrams and the relationship of polynomials to whole numbers to add and subtract polynomials with like terms

Basic: Add and subtract polynomials and multiply polynomials by constants, both supported by manipulatives or visual models

Proficient: Multiply binomials

Accelerated: Multiply binomials by trinomials

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.CED.1, A.CED.3, A.CED.4	Essential Understanding	Vocabulary
Create equations that describe numbers or relationships. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. a. Focus on applying linear and simple exponential expressions. b. Focus on applying simple quadratic expressions. Simple exponent expressions include integer exponents only. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V=IR to highlight the resistance R, or rearrange the formula for the area of a circle $A = \pi r^2$ to highlight radius r.	Students should be able to create and solve equations in one variable to answer questions. Students should be able to interpret word problems and form expressions, equations and inequalities in order to solve a problem. They must be able to translate a word problem into an algebraic equation. Students need to be able to identify when a common formula is needed for the given context. **Extended Understanding** Students can start learning quadratic, rational, and exponential functions to address all aspects of this standard. Once students are familiar with these operations individually, they should be asked to distinguish them from each other.	- Coefficient - Equation - Exponential - Function - Inequality - Linear - Literal - Polynomial - Rational - System of equations - Variable

Essential Skills

- I can create linear, quadratic, rational and exponential equations and inequalities in one variable and use them in a contextual situation to solve problems.
- I can write and use a system of equations and/or inequalities to solve a real-world problem. I can recognize that the equations and inequalities represent the constraints of the problems.
- I can solve multi-variable formulas or literal equations for a specific variable.

Instructional Strategies

Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations in two or more variables that may involve quadratic, exponential or rational functions.

Discuss the importance of using appropriate labels and scales on the axes when representing functions with graphs. Examine real-world graphs in terms of constraints that are necessary to balance a mathematical model with the real-world context. For example, a student writing an equation to model the maximum area when the perimeter of a rectangle is 12 inches should recognize that $y=x\,(6 - x)$ only makes sense when $0 < x < 6$.

Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.

Use a graphing calculator to demonstrate how dramatically the shape of a curve can change when the scale of the graph is altered for one or both variables.

Common Misconceptions and Challenges

Students may believe that equations of linear, quadratic and other functions are abstract and exist only “in a math book,” without seeing the usefulness of these functions as modeling real-world phenomena.

Students believe that the labels and scales on a graph are not important and can be assumed by a reader, and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.

Performance Level Descriptors

Limited: N/A

Basic: Create equations and inequalities in one variable and use them to solve simple routine problems; solve simple linear equations and inequalities with integer coefficients and coefficients represented by letters, including formulas; evaluate given possible solutions as viable or non-viable options in a modeling context

Proficient: Create exponential equations in one variable and use them to solve routine problems; select a viable argument to justify a solution method for a simple linear equation

Accelerated: Create quadratic and exponential equations and inequalities in one variable and use them to solve routine problems

Advanced: Create quadratic and exponential equations and inequalities in one variable and use them to accurately solve routine and non-routine problems; find and interpret solutions as viable or non-viable options in a modeling context

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.CED.2, A.CED.3, A.CED.4

Create equations that describe numbers or relationships.

Create equations in two or more variables to represent relationships between quantities; graph an equation on coordinate axes with labels and scales. Focus on applying linear, quadratic and simple exponential expressions.

Simple exponent expressions includes integer exponents only

Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Focus on formulas in which the variable of interest is linear or square. For example, rearrange OHM's law V=IR to highlight the resistance R, or rearrange the formula for the area of a circle $\boldsymbol{A = \pi r^2}$ to highlight radius r.

Essential Understanding

Students should be able to accurately represent the linear relationship between quantities in real-world situations as an equation and in a graph.

Extended Understanding

Students should be provided additional opportunities to work with/master the modeling cycle through problems that can be solved using equations and inequalities in one variable, systems of equations, and graphing.

Vocabulary

- Bounded solution
- Consistent system
- Constraints
- Dependent system
- Elimination
- Feasible region
- Inconsistent system
- Independent system
- Linear combination
- Linear inequality
- Literal
- Maximum
- Minimum
- Ordered pair
- Unbounded solution
- Vertex

Essential Skills

- I can create equations in two or more variables to represent relationships between quantities.
- I can graph equations in two variables on a coordinate plane and label the axes and scales.
- I can write and use a system of equations and/or inequalities to solve a real-world problem
- I can recognize that the equations and inequalities represent the constraints of the problems.
- I can solve multi-variable formulas or literal equations, for a specific variable.

Instructional Strategies

Provide opportunities for students to practice writing /generating a constraint equation for a given context.

Common Misconceptions and Challenges

Many students are confused in knowing which method of solving a system of equations is the best, and why. Some students may lack comprehension about the real-life constraints on variables.

Students can encounter difficulties in using elimination to solve systems; students must find the least common multiple of one variable, and remember to distribute completely if a multiple of an equation is needed.

Some students only solve for one variable instead of all variables, or answering the question that is being asked.

Some students substitute a value for a variable that is either incorrect, or they substitute the value into the wrong variable.

When solving with elimination or substitution, students do not understand how to recognize the special cases that result in infinite solutions or no solutions. When graphing linear inequalities, students do not always use the appropriate solid line (≤ and ≥) or dotted line (< and >) or shade above or below the line correctly.

Students do not know how to model a word problem mathematically with an inequality, especially when there is more than one inequality that must be extrapolated from the information given in the problem.

Students do not understand that an inequality application may have many solutions, not just one.

Performance Level Descriptors

Limited: N/A

Basic: Solve routine quadratic equations with integer solutions

Proficient: Create exponential equations in two variables and use them to solve routine problems

Accelerated: Create quadratic and exponential equations and inequalities in two variables and use them to solve routine problems

Advanced: Create quadratic and exponential equations and inequalities in one or two variables and use them to accurately solve routine and non-routine problems

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.1 **Understand solving equations as a process of reasoning and explain the reasoning.** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	**<u>Essential Understanding</u>** The foundation of this standard is found in the Mathematical Practices: MP1- Make sense of problems and persevere in solving them; MP2- Reason abstractly and quantitatively; MP 3 -Construct viable arguments and critique the reasoning of others. Problem solving, reasoning and proof, communication, representations and connections are "processes and proficiencies" with longstanding importance for all students of mathematics **<u>Extended Understanding</u>** Students should focus on and master this standard for linear equations so that they can extend and apply their reasoning to other types of equations (i.e., radical equations) in future courses.	**<u>Vocabulary</u>** - Associative - Constant - Distribute - Equal - Equivalent - Inverse - Isolate the variable - Order of operation - Unknown - Variable

Essential Skills

- I can, assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process; justifications may include the associative, commutative, and division properties, combining like terms, multiplication by 1, etc.

Instructional Strategies Challenge students to justify each step of solving an equation. Transforming 2x - 5 = 7 to 2x= 12 is possible because 5 = 5, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended much like providing evidence for steps of a geometric proof. Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation; the order of steps taken will not matter. More advanced students may work with solving both simple rational and radical equations that have no extraneous solutions before moving on to equations that result in quadratics and possible solutions that need to be eliminated. It is very important that students can reason how and why extraneous solutions arise. Provide visual examples of radical and rational equations with technology so that students can see the solution as the intersection of two functions and further understand how extraneous solutions do not fit the model.
Common Misconceptions and Challenges Students may believe that solving an equation such as 3x +1=7 involves "only removing the 1," failing to realize that the equation 1 = 1 is being subtracted to produce the next step. Additionally, students may believe that all solutions to radical and rational equations are viable without recognizing that there are times when extraneous solutions are generated and must be eliminated. Some students may not be comfortable with the order of operations. Stress to the students that now they will be working the order of operations "backwards" to find the value of the unknown.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: N/A

Accelerated: Construct a viable argument to justify a solution method for a system of linear equations

Advanced: Construct a viable argument to justify a solution method for a quadratic equation

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.3

Solve equations and inequalities in one variable.

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

<u>Essential Understanding</u>

Students should be able to reason and apply the four basic operations to both sides of an equation/inequality.

Students should be able to use and understand steps involved in equation/inequality solving to analyze the errors in their work and the work of others

<u>Extended Understanding</u>

Students will eventually need to understand that the application of properties is also appropriate when working with equations or inequalities that include more than one variable, fractions and decimals. Regardless of the type of numbers or variables in the equation or inequality, students will need to examine the validity of each step in the solution process.

<u>Vocabulary</u>

- Coefficient
- Constant
- Equivalent
- Inequality
- Isolate the variable
- Like terms
- Literal
- Order of operation
- PEMDAS
- Unknown
- Variable

Essential Skills

- I can solve linear equations in one variable, including coefficients represented by letters.
- I can solve linear inequalities in one variable, including coefficients represented by letters.

Instructional Strategies It may be beneficial to remind students of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences or multiplying both sides of an equation or inequality by the common denominator of the fractions. Students must be aware of what it means to check an inequality's solution. The substitution of the end points of the solution set in the original inequality should give equality regardless of the presence or the absence of an equal sign in the original sentence. The substitution of any value from the rest of the solution set should give a correct inequality. Solving equations for the specified letter with coefficients represented by letters is similar to solving an equation with one variable. Provide students with an opportunity to abstract from particular numbers and apply the same kind of manipulations to formulas as they did to equations. Draw students' attention to equations containing variables with subscripts. The same variables with different subscripts should be viewed as different variables that cannot be combined as like terms.
Common Misconceptions and Challenges Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (for ex., $3x > -15$ or $x < -5$). Some students may believe that subscripts can be combined as $b_1 + b_2 = b_3$ *and* the sum of different variables *d* and *D* is *2D* (*d* +D = 2D). Some students may think that rewriting equations into various forms (taking square roots, completing the square, using quadratic formula and factoring) are isolated techniques within a unit of quadratic equations. Teachers should help students see the value of these skills in the context of solving higher degree equations and examining different families of functions.

Performance Level Descriptors

Limited: Solve simple linear equations with integer coefficients and inequalities with whole number coefficients in one variable situations, with integer constants and whole number solutions

Basic: N/A

Proficient: Solve multi-step linear equations and inequalities with integer coefficients in one variable situations; solve multi-step linear equations with coefficients represented by letters, including formulas

Accelerated: Solve multi-step linear equations and inequalities with rational coefficients in one variable situations; solve multi-step linear equations with coefficients represented by letters, including formulas, which could include factoring or distributive property

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.4	Essential Understanding	Vocabulary
Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(\mathbf{x-p})^2 = \mathbf{q}$ that has the same solutions. ~~Derive the quadratic formula from this form.~~ (This is no longer required) b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $\mathbf{x^2 = 49}$; taking square roots; completing the square; applying the quadratic; or utilizing the Zero-Product Property after factoring.	- Students will be able to complete the square. - Students should be able to choose from multiple methods for solving quadratic equations the best method for each problem. **Extended Understanding** - Students should be shown how completing the square can be used to derive the quadratic formula itself.	- Complete the square - Derive - Inspection - Quadratic equation - Recognize - Solve - Transform - Variable

Essential Skills

- I can use the method of completing the square to transform any quadratic equation in x into an equation of the form (x-p)2 = q that has the same solutions.
- I can solve quadratic equations in one variable.
- I can derive the quadratic formula by completing the square on a quadratic equation in x.
- I can solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring.
- I can recognize when the quadratic formula gives complex solutions.

Instructional Strategies

Completing the square is usually introduced for several reasons: to find the vertex of a parabola whose equation has been expanded; to look at the parabola through the lenses of translations of a “parent” parabola $y = x^2$; and to derive a quadratic formula. Completing the square is a very useful tool that will be used repeatedly by students in many areas of mathematics. Teachers should carefully balance traditional paper-pencil skills of manipulating quadratic equations and properties of their graphs.

Start by inspecting equations such as $x^2 = 9$ that has two solutions, 3 and -3. Next, progress to equations such as $(x-7)^2 = 9$ by substituting $x-7$ for x and solving them either by “inspection” or by taking the square root on each side:

$$x - 7 = 3 \qquad \text{and} \qquad x - 7 = -3$$

$$x = 10 \qquad\qquad\qquad x = 4$$

Graph both pairs of solutions (-3 and 3, 4 and 10) on the number line and notice that 4 and 10 are 7 units to the right of -3 and 3. So, the substitution of $x-7$ for x moved the solutions 7 units to the right. Next, graph the function $y = (x-7)^2 - 9$, pointing out that the x-intercepts are 4 and 10, and emphasizing that the graph is the translation of 7 units to the right and 9 units down from the position of the graph of the parent function $y = x^2$ that passes through the origin. Generate more equations of the form $y = a(x-h)^2 + k$ and compare their graphs using a graphing technology.

Discourage students from giving a preference to a particular method of solving quadratic equations. Students need experience in analyzing a given problem to choose an appropriate solution method before their computations become burdensome.

Common Misconceptions and Challenges

Some students may believe that for equations containing fractions only on one side, it requires “clearing fractions” (the use of multiplication) only on that side of the equation. To address this misconception, start by demonstrating the solution methods for equations similar to $\frac{1}{4}x + \frac{1}{5}x + \frac{1}{6}x + 46 = x$ and stress that the Multiplication Property of Equality is applied to both sides, which are multiplied by 60.

Students may confuse the rule of changing a sign of an inequality when multiplying or dividing by a negative number with changing the sign of an inequality when one or two sides of the inequality become negative (e.g., $3x > -15$ or $x < -5$).

Some students may believe that subscripts can be combined as $b_1 + b_2 = b_3$ and the sum of different variables d and D is 2D ($d + D = 2D$).

Some students may think that rewriting equations into various forms (taking square roots, completing the square, using quadratic formula and factoring) are isolated techniques within a unit of quadratic equations. Teachers should help students see the value of these skills in the context of solving higher degree equations and examining different families of functions.

Performance Level Descriptors

Limited: Find square roots of perfect squares

Basic: N/A

Proficient: Solve quadratic equations with integer coefficients and constants by a) factoring or quadratic formula, where solutions may be rational, and b) completing the square where a = 1

Accelerated: Solve quadratic equations with rational coefficients and constants by factoring or completing the square

Advanced: Choose an appropriate method to solve a quadratic equation, according to the initial form of the equation, which could include simplifying initial expressions and with possible complex solutions

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.5, A.REI.6

Solve systems of equations.

Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solution.

Solve systems of linear equations algebraically and graphically.

a. Limit to pairs of linear equations in two variables.

Essential Understanding

Solving a system of linear equations means finding the point at which the two (or more lines intersect. This happens when the same set of *x* and *y* values satisfy all the linear equations in the system.

Extended Understanding

Students may use matrices to solve systems of linear equations if students are comfortable with/have mastered finding solutions using previously learned methods

Vocabulary

- Algebraically
- Bounded
- Consistent
- Dependent system
- Graphically
- Independent system
- Inequality
- Linear combination
- Linear inequality
- Ordered pair
- Solution
- Substitution
- System of linear equations
- Unbounded
- Unknown

Essential Skills

- I can solve systems of equations using the elimination method (sometimes called linear combinations).
- I can recognize and use properties of equality to maintain equivalent systems of equations.
- I can solve a system of equations by substitution (solving for one variable in the first equation and substituting it into the second equation).
- I can solve systems of equations using graphs.

Instructional Strategies

Provide students opportunities to practice linear vs. non-linear systems; consistent vs. inconsistent systems; pure computational vs. real-world contextual problems (e.g., chemistry and physics applications encountered in science classes). A rich variety of examples can lead to discussions of the relationships between coefficients and consistency that can be extended to graphing and later to determinants and matrices.

Provide examples of real-world situations that can be solved by writing systems of equations or making matrices and have students explore the graphs of the equations on the calculator to determine the relevancy of the graphs and units to the problem context.

Common Misconceptions and Challenges

Some students are challenged trying to determine which method of solving a system of equations is the best, and why. Many students lack understanding about the real-life constraints on variables.

When using elimination to solve systems, students must find the least common multiple of one variable, and remember to distribute completely if a multiple of an equation is needed.

Some students only solve for one variable instead of all variables, or answering the question that is being asked.

When solving with elimination or substitution, students do not understand how to recognize the special cases that result in infinite solutions or no solutions.

Performance Level Descriptors

Limited: Solve linear equations in two variables to describe a familiar situation using whole numbers supported by algebra manipulatives or diagrams

Basic: Solve simple systems of two linear equations in two variables exactly algebraically

Proficient: N/A

Accelerated: N/A

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle $\mathbf{x^2 + y^2 = 3}$.	**Essential Understanding** - Students should be able to solve systems by using algebra, as well as making graphs and looking at intersection points. **Extended Understanding** - Students can use technology such as graphing calculators or online sites such as Desmos.com to quickly create different types of graphs at the same time.	**Vocabulary** - Correspondence (between) - Explain - Linear equation - Point of intersection - Quadratic equation - Solve - System - Transform

Essential Skills

- I can solve simple systems consisting of linear, quadratic, and circular equations.
- I can transform a simple system consisting of a linear equation and quadratic equation in 2 variables so that a solution can be found algebraically and graphically.
- I can explain the correspondence between the algebraic and graphical solutions to a simple system consisting of a linear equation and a quadratic equation in 2 variables.

Instructional Strategies

Show students how to use substitution to solve a system consisting of a linear and a quadratic equation in two variables. Use tools like www.desmos.com, www.geogebra.org or graphing calculators to find points of intersection by inspection.

Common Misconceptions and Challenges

Some of the algebra gets tricky for students. Watch closely to ensure mistakes are not made, particulary when doing inverse operations and using substitution.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: N/A

Accelerated: Solve a system consisting of a linear equation and a quadratic equation in two variables graphically, and algebraically in simpler cases

Advanced: Accurately and efficiently solve a system consisting of a linear equation and a quadratic equation in two variables algebraically

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.10, A.REI.12 **Represent and solve equations and inequalities graphically.** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	**Essential Understanding** Students should know that equations actually mean something visually; students should know that an equation with two variables all have an *x* and a *y,* meaning that instead of having an equation with one variable (and therefore one solution), we can have many different solutions; graphically, we can represent these solutions by drawing a curve or line through all the pairs of solutions (one for *x* and one for *y*) that work for that particular equation. Students should know how to graph a linear inequality; students should be able to Identify characteristics of a linear inequality and a system of linear inequalities, such as: boundary line, shading, and determine the appropriate points to test and derive a solution set from. **Extended Understanding** Students can explore nonlinear situations to see that even complex equations that might involve quadratics, absolute value, or rational functions can be solved fairly easily using graphing methods and technology.	**Vocabulary** - Bounded - Feasible region - Graphing - Half plane - Linear inequality - Linear programming - Maximum - Minimum - Plane - Variable - Vertex

Essential Skills

- I can understand that all solutions to an equation in two variables are contained on the graph of that equation.
- I can graph the solutions to a linear inequality in two variables as a half-plane, excluding the boundary for non-inclusive inequalities.
- I can graph the solution set to a system of linear inequalities in two variables as the intersection of their corresponding half-planes.

Instructional Strategies

Students should be able prove that the equations and their graphs correspond to one another. pick the numbers -1, 0, and 3 for *x*. Looking at the equation, $7x – 18 = y$, taking a point on the line that is easily identifiable, say (2, -4), and plug the values into the equation, $-4 = 7(2) – 18$, which simplifies to $-4 = -4$. That way, students will be sure that points on the line or curve are on the graph.

It's also important to prove the opposite. For example, the coordinate (4, 1), which is not on the line, is also not a solution to our equation. If we plug in the coordinates, we can confirm this: $1 = 7(4) – 18$ is false because $1 \neq 10$. This means (4, 1) isn't a solution to our equation and not a point on the line.

Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

Graphing calculators or computer software that generates graphs and tables for solving equations can be utilized. Using techn ology, have students graph a function and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation.

Common Misconceptions and Challenges

When graphing linear inequalities, students do not always use the appropriate solid line (≤ and ≥) or dotted line (< and >) o r shade above or below the line correctly.

Students do not know how to model a word problem mathematically with an inequality, especially when there is more than one inequality that must be extrapolated from the information given in the problem

Students do not understand that an inequality application may have many solutions, not just one.

Assuming students know how to graph a linear equation (by table or by putting the inequality in slope-intercept form) and they overcome the struggle of shading on a single inequality, students may still have trouble determining the solution to a system where a student must identify what area of the graph satisfies every inequality in the system.

Performance Level Descriptors

Limited: N/A

Basic: Graph the solutions to a linear inequality in two variables as a half-plane

Proficient: Graph equations on coordinate axes with appropriate labels and scales; understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line); solve a system of linear inequalities in two variables graphically

Accelerated: N/A

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

A.REI.11

Represent and solve equations and inequalities graphically.

Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make the tables of values, or find successive approximations.

Essential Understanding

Studentswill be expected to determine if a given ordered pair is a solution to a given system of linear equations by evaluating both equations/inequalities.

Students will be expected to approximate the solution to the system with a graph (manually and with graphing technology).

Extended Understanding

Students should be able to model the reasoning process when discussing and /or graphing equations and inequalities. Students should have opportunities to work with cases involving absolute value, exponential, and logarithmic functions.

Vocabulary

- Absolute value
- Coefficient
- Equation
- Exponential
- Function
- Inequality
- Linear
- Literal
- Logarithmic
- Polynomial
- Rational
- System of equation
- System of inequality
- Variable

Essential Skills

- I can explain why the intersection of y=f(x) and y=g(x) is the solution of f(x)=g(x) for any combination of linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Find the solutions(s) by:
  - Using technology to graph the equations and determine their point of intersection,
  - Using tables of values, or
  - Using successive approximations that become closer and closer to the actual value.

Instructional Strategies

Beginning with simple, real-world examples, help students to recognize a graph as a set of solutions to an equation. For example, if the equation y = 6x + 5 represents the amount of money paid to a babysitter (i.e., $5 for gas to drive to the job and $6/hour to do the work), then every point on the line represents an amount of money paid, given the amount of time worked.

Explore visual ways to solve an equation such as $2x + 3 = x – 7$ by graphing the functions $y = 2x + 3$ and $y = x – 7$. Students should recognize that the intersection point of the lines is at (-10, -17). Students should be able to verbalize that the intersection point means that when x = -10 is substituted into both sides of the equation, each side simplifies to a value of -17. Therefore, -10 is the solution to the equation. This same approach can be used whether the functions in the original equation are linear, nonlinear or both.

Using technology, have students graph a function and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation.

Common Misconceptions and Challenges

Students may believe that the graph of a function is simply a line or curve “connecting the dots,” without recognizing that the graph represents all solutions to the equation.

Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions.

Students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

Performance Level Descriptors

Limited: N/A

Basic: Understand that the graph of a function *f* is the graph of the equation $y = f(x)$

Proficient: N/A

Accelerated: Find the approximate solutions of the equation $f(x) = g(x)$ by finding the x-coordinates where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect, including cases where f(x) and/or g(x) are linear or exponential

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.IF.1, F.IF.2, F.IF. 3

Understand the concept of a function and use function notation.

Understand that a function from one set (domain) to another set (range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, the f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f (0) = f (1) = 1, f (n+1) = f (n) +f (n-1) for n> 1.

Essential Understanding

- Students must be able to understand the concept of a function and use function notation.

- Students will be required to analyze functions in terms of a specific context.

Extended Understanding

- Students will be expected to understand that relations and functions can be represented numerically, graphically, algebraically, and/or verbally.

- Students will be expected to move flexibly between the different representations of the same function for comparison.

Vocabulary

- Corresponding
- Domain
- Element
- Evaluate
- Function
- Input
- Mapping
- Notation
- Ordered pairs
- Output
- Range
- Rate of change
- Recursive
- Relation
- Set
- Subset
- Vertical line test

Essential Skills

- I can use the definition of a function to determine whether a relationship is a function given a table, graph or words.
- I can identity *x* as an element of the domain (input) and *f(x)* as an element in the range (output) when given the function *f(x).*
- I can recognize the graph of the function, *f*, is the graph of the equation *y=f(x).*
- I can use *f(x)* notation when a relation is determined to be a function.
- I can evaluate functions for inputs in their domain.
- I can interpret statements that use function notation in terms of the context in which they are used.

Instructional Strategies

Provide students with many examples of functional relationships, both linear and non-linear. Use real-world examples, such as the growth of an investment fund over time, so that students can not only describe what they see in a table, equation, or graph, but also can relate the features to the real-life meanings.

Allow students to collect their own data sets such as the falling temperature of a glass of hot water when removed from a flame versus the amount of time to generate tables and graphs for discussion and interpretation.

Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers.

Common Misconceptions and Challenges

Students may believe that all relationships having an input and an output are functions and therefore misuse the function terminology.

Students may also believe that the notation $f(x)$ means to multiply some value f times another value x. The notation alone can be confusing and needs careful development. For example, $f(2)$ means the output value of the function f when the input value is 2.

Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula. Students naturally tend to look "down" a table to find the pattern but need to realize that finding the 100th term requires knowing the 99th term unless an explicit formula is developed.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: N/A

Accelerated: N/A

Advanced: Interpret statements that use function notation in terms of a context

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.IF.4, F.IF.5, F.IF.6	**Essential Understanding**	**Vocabulary**
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Focus on linear, quadratic, and exponential functions. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Focus on linear, quadratic, and exponential functions. 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	- Students understand how to interpret domain and range from a graph. - Students understand how to interpret domain and range from a real-world context. - Students understand how to calculate average rate of change of a function and understand why it is the slope between two points. **Extended Understanding** - Students can think about how they would try to calculate instantaneous rate of change.	- Average rate of change - Calculate - End behavior - Global max & min - Intercepts - Interpret - Local/relative max & min - Model - Periodicity

Essential Skills

- I can define and recognize the key features in tables and graphs of linear and exponential functions: intercepts; intervals where the function is increasing, decreasing, positive, or negative, and end behavior.
- I can identify whether the function is linear or exponential, given its table or graph.
- I can interpret key features of graphs and tables of function in the terms of the contextual quantities the function represents.
- I can sketch graphs showing key features of a function that models a relationship between two quantities from a given verbal description of the relationship.
- I can explain why a domain is appropriate for a given situation.
- I can recognize slope as an average rate of change.

Instructional Strategies Flexibly move from examining a graph and describing its characteristics (e.g., intercepts, relative maximums, etc.) to using a set of given characteristics to sketch the graph of a function. Examine a table of related quantities and identify features in the table, such as intervals on which the function increases, decreases, or exhibits periodic behavior. Recognize appropriate domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers. Given a table of values, such as the height of a plant over time, students can estimate the rate of plant growth. Also, if the relationship between time and height is expressed as a linear equation, students should explain the meaning of the slope of the line. Finally, if the relationship is illustrated as a linear or non-linear graph, the student should select points on the graph and use them to estimate the growth rate over a given interval.
Common Misconceptions and Challenges Students may believe it is reasonable to input any x-value into a function, so they will need to examine multiple situations in which there are various limitations to the domain. Students may also believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning, and the idea of a rate of change is fundamental to understanding major concepts from geometry to calculus.

Performance Level Descriptors

Limited: Given a graph of a simple function modeling a linear relationship between two quantities, determine where the function is increasing, decreasing, positive or negative; calculate the average rate of change (slope) of linear functions given tables and/or graphs

Basic: Given a graph of a function that models a linear relationship between two quantities, interpret key features: intercepts; intervals where the function is increasing, decreasing, positive or negative; Calculate the average rate of change (slope) of a linear function as a table over a specified interval

Proficient: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities; sketch graphs showing key features given a verbal description of the relationship; relate the domain of a function to its graph; calculate the average rate of change (slope) of a linear function presented symbolically and/or graphically over a specified interval

Accelerated: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities; sketch graphs showing key features given a verbal description of the relationship; relate the domain of a function to the quantitative relationship it describes; interpret the average rate of change of a function over a specified interval

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.IF.7-9	Essential Understanding	Vocabulary
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. a. Graph linear functions and indicate intercepts. b. Graph quadratic functions and indicate intercepts, maxima, and minima. e. Graph simple exponential functions, indicating intercepts and end behavior. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a.i. Focus on completing the square to quadratic functions with the leading coefficient of 1. b.i. Focus on exponential functions evaluated at integer inputs. 9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). b. Focus on linear, quadratic, and exponential functions.	- Students can interpret key features of graphs of quadratic functions in a real-world context. -Students can complete the square to determine the vertex of a quadratic function. -Students can factor a quadratic equation to determine the x-intercepts -Students can solve quadratics using factoring and completing the square. **Extended Understanding** - Students can write a quadratic equation given a table of values or a quadratic pattern.	- Compare - Complete the square - Factor - Interpret - Quadratic equation - Quadratic function - Standard form - Vertex form

Essential Skills

- I can graph exponential functions by hand in simple cases or using technology for more complicated cases, and show intercepts and end behavior
- I can determine the difference between simple and complicated linear and exponential functions and know when the use of technology is appropriate
- I can differentiate between exponential and linear functions using a variety of descriptors (graphically, verbally, numerically, and algebraically)
- I can use a variety of function representations algebraically, graphically, numerically in tables, or by verbal descriptions) to compare and contrast properties of two functions

Instructional Strategies

Explore various families of functions and help students to make connections in terms of general features. For example, just as the function $y = (x + 3)^2 - 5$ represents a translation of the function $y = x$ by 3 units to the left and 5 units down, the same is true for the function $y = |x + 3| - 5$ as a translation of the absolute value function $y = |x|$.

Discover that the factored form of a quadratic or polynomial equation can be used to determine the zeros, which in turn can be used to identify maxima, minima and end behaviors.

Use various representations of the same function to emphasize different characteristics of that function. For example, the y-intercept of the function $y = x^2 - 4x - 12$ is easy to recognize as $(0, -12)$. However, rewriting the function as $y = (x - 6)(x + 2)$ reveals zeros at $(6, 0)$ and at $(-2, 0)$. Furthermore, completing the square allows the equation to be written as $y = (x - 2)^2 - 16$, which shows that the vertex (and minimum point) of the parabola is at $(2, -16)$.

Examine multiple real-world examples of exponential functions so that students recognize that a base between 0 and 1 (such as an equation describing depreciation of an automobile [$f(x) = 15{,}000(0.8)^x$ represents the value of a $15,000 automobile that depreciates 20% per year over the course of x years]) results in an exponential decay, while a base greater than 1 (such as the value of an investment over time [$f(x) = 5{,}000(1.07)^x$ represents the value of an investment of $5,000 when increasing in value by 7% per year for x years]) illustrates growth.

Common Misconceptions and Challenges

Students may believe that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphjs.

Students may also believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials, and that they will come to understand the usefulness of these skills in the context of examining characteristics of functions.

Additionally, students may believe that the process of rewriting equations into various forms is simply an algebra sumbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

Performance Level Descriptors

Limited: Graph linear functions and show whole number intercepts; match graphs of linear equations to tables of solutions

Basic: Recognize the difference between a linear and exponential situation represented by a graph or equation

Proficient: Recognize the difference between linear and exponential situations from real-world contexts or a variety of representations; graph quadratic functions and show intercepts, maxima and minima; given two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions), compare the properties of the two functions

Accelerated: Graph exponential functions, showing end behavior; interpret zeros, extreme values, and symmetry of the graph of a quadratic function in terms of a context; use the process of completing the square in a quadratic function, where a = 1, to show extreme values and symmetry of the graph

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.IF.7, F.IF.7a, F.IF.9	Essential Understanding	Vocabulary
Analyze functions using different representations. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear functions and indicate intercepts. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. b. Focus on linear, quadratic, and exponential functions.	- Students will be expected to master flexible movement between the multiple representations. - Students are expected to increase comfort level in understanding other representations mentally even when only one representation is given. **Extended Understanding** - Students can better understand the characteristics of representations by studying the eight major families of functions.	- Constant - Domain - Exponential - Linear - Parent function - Rate of change - Sequence - Slope - Standard form - X-intercept - Y-intercept

Essential Skills

- I can graph functions expressed symbolically and show key features of the graph, graphing simple cases by hand and using technology to show more complicated cases.
- I can write a function in equivalent forms to show different properties of the function.
- I can explain the different properties of a function that are revealed by writing a function in equivalent forms.
- I can compare and contrast features of two functions each represented in a different way.

Instructional Strategies

Graphing utilities on a calculator and/or computer can be used to demonstrate the changes in behavior of a function as various parameters are varied. Add families of functions, one at a time, to the students' knowledge base so they can see connections among behaviors of the various functions.

Provide numerous examples of real-world contexts such as exponential growth and decay situations (e.g., a population that declines by 10% per year) to help students apply an understanding of functions in context.

Common Misconceptions and Challenges

Students may believe that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs.

Students may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise rather than serving a purpose of allowing different features of the function to be exhibited.

Performance Level Descriptors

Limited: Graph linear functions and show whole number intercepts; match graphs of linear equations to tables of solutions

Basic: Recognize the difference between a linear and exponential situation represented by a graph or equation

Proficient: Recognize the difference between linear and exponential situations from real-world contexts or a variety of representations; given two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions), compare the properties of the two functions

Accelerated: Graph exponential functions, showing end behavior

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.BF.1, F.BF.2 — Build a function that models.	Essential Understanding	Vocabulary
Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. Focus on linear and exponential functions, and situations that exhibit quadratic or exponential relationships. Write arithmetic sequences both recursively and with an explicit formula use them to model situations, and translate between the two forms.	Examination of functions is extended to include recursive and explicit representations and sequences of numbers that may not have a linear relationship. **Extended Understanding** Using a variety of functions (e.g., linear, exponential, constant, students can increase understanding of the different representations by representing functions as a set of ordered pairs, a table, a graph, and an equation.	- Arithmetic sequence - Composite functions - Correspondence - Direct variation - Explicit formula - Function - Geometric sequence - Inverse function - Inverse relationship - Quantities - Recursive

Essential Skills

- I can, from context, either write an explicit expression, define a recursive process, or describe the calculations needed to model a function between two quantities.
- I can combine standard function types, such as linear and exponential, using arithmetic operations. I can compose functions.
- I can write arithmetic sequences recursively and explicitly, use the two forms to model a situation and translate between the two forms.
- I can write geometric sequences recursively and explicitly, use the two forms to model a situation, and translate between the two forms.
- I can understand that linear functions are the explicitly form of recursively-defined arithmetic sequences and that exponential functions are the explicit form of recursively-defined geometric sequences.

Instructional Strategies

Provide a real-world example (e.g., a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking "down" the table to describe a recursive relationship, as well as "across" the table to determine an explicit formula to find the distance traveled if the number of minutes is known.

Write out terms in a table in an expanded form to help students see what is happening. For example, if the *y*-values are 2, 4, 8, 16, they could be written as 2, so that students recognize that 2 is being used multiple times as a factor.

Focus on one representation and its related language – recursive or explicit – at a time so that students are not confusing the formats.

Provide examples of when functions can be combined, such as determining a function describing the monthly cost for owning two vehicles when a function for the cost of each (given the number of miles driven) is known.

Common Misconceptions and Challenges

Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula. Students naturally tend to look "down" a table to find the pattern but need to realize that finding the 100th term requires knowing the 99thterm unless an explicit formula is developed.

Students may also believe that arithmetic and geometric sequences are the same. Students need experiences with both types of sequences to be able to recognize the difference and more readily develop formulas to describe them.

Advanced students who study composition of functions may misunderstand function notation to represent multiplication (e.g., $f(g(x))$ means to multiply the f and g function values).

When studying functions, students sometimes interchange the input and output values. This will lead to confusion about domain and range, and determining if a relation is a function. This can also interfere with a student being able to find the appropriate inverse function, or the correct equation to model a relationship between two quantities.

Performance Level Descriptors

Limited: Given a straightforward linear relationship in context, write a function

Basic: N/A

Proficient: N/A

Accelerated: Create an explicit function to define an arithmetic sequence

Advanced: Recognize a recursive function that defines a sequence from a context

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.BF.3, F.BF.4 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. a. Focus on transformations of graphs of quadratic functions, except for $\boldsymbol{f(kx)}$. 4. Find inverse functions. a. Informally determine the input of a function when the output is known.	**Essential Understanding** - Students understand how to transform graphs of parent functions. -Students understand how to describe transformations of parent functions including how to use the proper notation. -Students understand how to find inverses of functions. **Extended Understanding** - Students find inverses of more complex functions.	**Vocabulary** - Dependent variable - Dilation/stretch - Direct variation - Experiment - Find - Function - Identify - Include - Independent variable - Inverse - Inverse function - Invertible - Noninvertible - Translation/shift

Essential Skills

- I can identify the effect a single transformation will have on the function (symbolic or graphic).
- I can use technology to identify effects of single transformations on graphs of functions.
- I can graph a given function by replacing f(x) with f(x)+k, kf(x), f(kx), or f(x+k) for specific values of k (both positive and negative).
- I can describe the differences and similarities between a parent function and the transformed function.
- I can recognize even and odd functions from their graphs and from their equations.

Instructional Strategies

Use graphing calculators or computers to explore the effects of a constant in the graph of a function. For example, students should be able to distinguish between the graphs of $y = x^2, y = 2x^2, y = x^2 + 2, y = (2x)^2$, and $y = (x + 2)^2$. This can be accomplished by allowing students to work with a single parent function and examine numerous parameter changes to make generalizations.

Distinguish between even and odd functions by providing several examples and helping students to recognize that a function is even if $f(-x) = f(x)$ and is odd if $f(-x) = -f(x)$. Visual approaches to identifying the graphs of even and odd functions can be used as well.

Provide examples of inverses that are not purely mathematical to introduce the idea. For example, given a function that names the capital of a state, $\mathrm{f(Ohio)} = \mathrm{Columbus}$, the inverse would be to input the capital city and have the state be the output, such that $f^{-1}(Denver) = Colorado$.

Common Misconceptions and Challenges

Students may believe that the graph of $y = (x - 4)^3$ is the graph of $y = x^3$ shifted 4 units to the left (due to the subtraction symbol). Examples should be explored by hand and on a graphing calculator to overcome this misconception.

Students may also believe that *even* and *odd* functions refer to the exponent of the variable, rather than the sketch of the graph and the behavior of the function.

Additionally, students may believe that all functions have inverses and need to see counterexamples as well as examples in which a non-invertible function can be made into an invertible function by restricting the domain. For example, $f(x) = x^2$ has inverse $(f^{-1}(x) = \sqrt{x})$ provided that the domain is restricted to $x \geq 0$.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: N/A

Accelerated: N/A

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.LE.1, F.LE.2, F.LE.3	Essential Understanding	Vocabulary
Construct and compare linear, quadratic, and exponenetial models and solve problems. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Show that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table). Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically.	Linear and exponential functions compare in their ability to predict real world events. Both functions can be described as having initial values and constant rates of change. **Extended Understanding** Changes in graphs are explored in more depth and the idea of functions having inverses is introduced. Advanced students can also expand their catalog of functions to include logarithmic cases.	- Arithmetic sequence - Constant - Decay - Decreasing - Exponential differences - Family of functions - Geometric sequence - Increasing - Initial value - Interval - Linear - Percent of change - Polynomial - Quadratic - Rate of change - Relation - Relative

Essential Skills

- I can, given a contextual situation, describe whether the situation in question has a linear pattern of change or an exponential pattern of change; I can show that linear functions change at the same rate over time and that exponential functions change by equal factors over time; I can describe situations where one quantity changes at a constant rate per unit interval as compared to another.
- I can create linear/exponential function given the following: arithmetic/geometric sequences, a graph, a description of a relationship, or two points read from a table.
- I can make the connection, using graphs and tables, that a quantity.

Instructional Strategies

Explore simple linear and exponential functions by engaging in hands-on experiments. For example, students can measure the diameters and related circumferences of several circles and determine a linear function that relates the diameter to the circumference – a linear function with a first common difference. They can then explore the value of an investment when told that the account will double in value every 12 years – an exponential function with a base of 2.

Compare tabular representations of a variety of functions to show that linear functions have a first common difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal *x*-intervals). Apply linear and exponential functions to real-world situations. For example, a person earning $10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.

Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns. Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and polynomial functions to show how the *y* (output) values of the exponential function eventually exceed those of polynomial functions.

Common Misconceptions and Challenges

Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

Students may also believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: N/A

Accelerated: N/A

Advanced: N/A

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

F.LE.5	**Interpret expressions for functions in terms of the situation they model.** Interpret the parameters in a linear or exponential function in terms of a context.	**Essential Understanding** - Students should be able to understand that functions model relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity -Limit exponential functions to those of the form $f(x) = b^x + k$. **Extended Understanding** Extended understanding should include opportunities for a deeper contextual understanding of parameter changes in both linear and exponential function situations.	**Vocabulary** - Decreasing - End behavior - Exponential - Expressions - Function - Increasing - Intercept - Interval - Linear - Maximum - Minimum - Model - Periodicity - Quantities - Rate of change - Relative - Slope - Symmetry

Essential Skills

- I can, based on the context of a situation, explain the meaning of the coefficients, factors, exponents, and/or intercepts in a linear or exponential function.

Instructional Strategies Resources can include graphing calculators or computer software that generates graphs and tables of functions. Use examples of real-world situations that apply linear and exponential functions to examine the effects of parameter changes. Identify web sites and other sources that provide raw data, such as the cost of products over time, population changes, etc. Provide students with opportunities to research raw data on the Internet (such as increases in gasoline consumption in China over the years) and to graph and generalize trends in growth, determining whether the growth is linear. Working in pairs or small groups, students can be given different parameters of a function to manipulate and compare the results to draw conclusions about the effects of the changes
Common Misconceptions and Challenges Students may believe that changing the slope of a linear function from “2” to “3” makes the graph steeper without realizing that there is a real-world context and reason for examining the slopes of lines. Similarly, an exponential function can appear to be abstract until applying it to a real-world situation involving population, cost, investments, etc. Exponential and linear functions can be decreasing functions. A decreasing exponential function does not need to have a negative sign in the expression: $y = (1/2)^{1/2}$, the percent increase only needs to be less than 100%. Small positive numbers represented in scientific notation are not negative (1.45^{-32} is not negative).

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: N/A

Accelerated: N/A

Advanced: Interpret the parameters in a linear or exponential function in terms of a context

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

S.ID.1, S.ID.2, S.ID.3

Summarize, represent, and interpret data on a single count of measurement variable.

Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model.

In the context of real-world applications using the GAISE model, use statistics appropriate to shape of the data distributions to compare center (median, mean) and spread (mean absolute deviation, interquartile range, standard deviation) of two or more different data sets.

In the context of real-world applications using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets accounting for possible effects of extreme data points (outliers).

Essential Understanding

- Students will reinforce their previous knowledge of data representation in the forms of histograms, tables, box-and-whisker plots and scatterplots.

- Students will study measures of variability (the way data is distributed), center and spread, including standard deviation, ---all used to compare and draw conclusions about sets of data.

- Students will analyze data that they will display and calculate with and without technology.

Extended Understanding

Students should understand that a statistical process is a problem-solving process consisting of four steps: formulating a question that can be answered by data; designing and implementing a plan that collects appropriate data; analyzing the data by graphical and/or numerical methods; and interpreting the analysis in the context of the original question.

Vocabulary

- Association
- Bivariate data
- Box plot
- Categorical data
- Causation
- Center
- Correlation coefficient
- Dot plot
- Frequency table
- Histogram
- Interquartile range
- Joint frequency
- Line of best fit
- Outlier
- Qualitative (numerical) variable
- Quantitative (numerical) variable
- Regression
- Residual
- Scatterplot
- Spread
- Standard deviation
- Symmetric
- Two-way frequency table

Essential Skills

- I can construct dot plots, histograms and box plots for data on a real number line.
- I can: describe a distribution using center and spread; I can use the correct measure of center and spread to describe a distribution that is symmetric or skewed; I can identify outliers (extreme data points) and their effects on data sets; I can compare two or more different data sets using the center and spread of each.
- I can interpret differences in different data sets in context; I can interpret differences due to possible effects of outliers.

Instructional Strategies

Opportunities should be provided for students to work through the statistical process. Now is a good time to investigate a problem of interest to the students and follow it through. The richer the question formulated, the more interesting is the process. Teachers and students should make extensive use of resources to perfect this very important first step. Global web resources can inspire projects. Although this domain addresses both categorical and quantitative data, there is no reference in the Standards 1 - 4 to categorical data. Note that Standard 5 in the next cluster (Summarize, represent, and interpret data on two categorical and quantitative variables) addresses analysis for two categorical variables on the same subject. To prepare for interpreting two categorical variables in Standard 5, this would be a good place to discuss graphs for one categorical variable (bar graph, pie graph) and measure of center (mode). Have students practice their understanding of the different types of graphs for categorical and numerical variables by constructing statistical posters. Note that a bar graph for categorical data may have frequency on the vertical (student's pizza preferences) or measurement on the vertical (radish root growth over time - days). Measures of center and spread for data sets without outliers are the mean and standard deviation, whereas median and interquartile ranges are better measures for data sets with outliers. Introduce the formula of standard deviation by reviewing the previously learned MAD (mean absolute deviation). The MAD is very intuitive and gives a solid foundation for developing the more complicated standard deviation measure. Informally observing the extent to which two boxplots or two dot plots overlap begins the discussion of drawing inferential conclusions. Don't shortcut this observation in comparing two data sets. As histograms for various data sets are drawn, common shapes appear. To characterize the shapes, curves are sketched through the midpoints of the tops of the histogram's rectangles. Of particular importance is a symmetric unimodal curve that has specific areas within one, two, and three standard deviations of its mean. It is called the Normal distribution and students need to be able to find areas (probabilities) for various events using tables or a graphing calculator.

Common Misconceptions and Challenges

Students believe: That a bar graph and a histogram are the same. A bar graph is appropriate when the horizontal axis has categories and the vertical axis is labeled by either frequency (e.g., book titles on the horizontal and number of students who like the respective books on the vertical) or measurement of some numerical variable (e.g., days of the week on the horizontal and median length of root growth of radish seeds on the vertical). A histogram has units of measurement of a numerical variable on the horizontal (e.g., ages with intervals of equal length); that the lengths of the intervals of a boxplot (min, Q1), (Q1, Q2), (Q2, Q3), (Q3, max) are related to the number of subjects in each interval. Students should understand that each interval theoretically contains one-fourth of the total number of subjects. Sketching an accompanying histogram and constructing a live boxplot may help in alleviating this misconception; that all bell-shaped curves are normal distributions. For a bell-shaped curve to be normal, there needs to be 68% of the distribution within one standard deviation of the mean, 95% within two, and 99.7% within three standard deviations.

Performance Level Descriptors

Limited: Describe the comparison of center (median, mean) of two different data sets

Basic: Represent given data in a different statistical model; interpret key features of a scatterplot (linear or nonlinear, correlation)

Proficient: Use statistics appropriate to the shape of the data distribution to compare center (median, mean) of two or more different data sets

Accelerated: Use statistics appropriate to the shape of the data distribution to compare spread (interquartile range, standard deviation) of two or more different data sets

Advanced: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

S.ID.5, S.ID.6a, S.ID.6c

Summarize, represent, and interpret data on two categorical and quantitative variables.

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
c. Fit a linear function for a scatter plot that suggests a linear association.

Essential Understanding

Students are expected to know that functions may be used to describe data and that if the data suggest a linear relationship, the relationship can be modeled with a regression line; its strength and direction can be expressed through a correlation coefficient.

Extended Understanding

Introduce technology in this section for ease of obtaining solutions or visualizing results. Many stats packages are available that will perform regression with multiple variables.

Vocabulary

- Bivariate data
- Categorical data
- Center
- Correlation coefficient
- Dot plot
- Frequency table
- Histogram
- Interquartile range
- Joint frequency
- Line of best fit
- Outlier
- Regression
- Residual
- Scatterplot
- Spread
- Standard deviation
- Symmetric
- Two-way frequency table

Essential Skills

- I can create a two-way table from two categorical variables and read values from a two-way table. Interpret joint, marginal, and relative frequencies in context. I can recognize associations and trends in data from a two-way table; I can recognize associations and trends in data from a two- way table; I can describe the form, strength and direction of the relationship; I can categorize data as linear or not; I can use algebraic methods and technology to fit a linear function to the data; I can use the function to predict values; I can explain the meaning of the slope and y- intercept in context;
- I can categorize data as exponential; I can use algebraic methods and technology to fit an exponential function to the data; I can use the function to predict values; I can explain the meaning of the growth rate and y-intercept in context; I can categorize data as quadratic; I can use algebraic methods and technology to fit a quadratic function to the data; I can use the function to predict values I can explain the meaning of the constant and coefficients in context; I can categorize data as linear or not; I can use algebraic methods and technology to fit a linear function to the data; I can use the function to predict values.

Instructional Strategies The focus is that two categorical or two quantitative variables are being measured on the same subject. In the categorical case, begin with two categories for each variable and represent them in a two-way table with the two values of one variable defining the rows and the two values of the other variable defining the columns. (Extending the number of rows and columns is easily done once students become comfortable with the 2x2 case.) The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values. Row totals and column totals constitute the marginal frequencies. Dividing joint or marginal frequencies by the total number of subjects define relative frequencies (and percentages), respectively. Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables. In the numerical or quantitative case, display the paired data in a scatterplot. Note that although the two variables in general will not have the same scale, e.g., total SAT versus grade-point average, it is best to begin with variables with the same scale such as SAT Verbal and SAT Math. Fitting functions to such data will avoid difficulties such as interpretation of slope in the linear case in which scales differ. Once students are comfortable with the same scale case, introducing different scales situations will be less problematic. Help students clearly distinguish between categorical and numerical variables by providing multiple examples of each type. Provide opportunities for students to formulate meaningful questions in the first step of the four-step process. This takes time and lots of practice leading to real-world contexts.
Common Misconceptions and Challenges **Outliers**: Incorrectly identifying an outlier without considering the context of the data first; drawing a conclusion about the center or spread of a data set when calculations have been influenced by an outlier; **Box plots:** Confusing the mean and median; Using the mean in a box plot instead of the median; Incorrectly setting up the number line before creating a dot plot or histogram; Forgetting to include data values on a dot plot or histogram because they are not labeled on the number line; Forgetting to put the data in numerical order first. Leaving some of the data out; Forgetting how to calculate a quartile that falls between two numbers; **Frequency Tables:** Incorrectly locating frequencies in the table; Incorrectly calculating conditional relative frequencies by being inconsistent in the method used (dividing by the number of times a response was given, the number of people with a given characteristic, or the total number of respondents); **Data Analysis**: Comparing graphical data that is not drawn using the same scale on the x- and /or y-axes; Comparing different measures of center or variation; Using an average, such as the mean, to compare data that has very small or very large data values; **Functions:** Confusing when to evaluate and when to solve a function. Using a linear function to estimate a relationship between two variables when an exponential function is a better fit; Stress that 'time' is usually along the x-axis; **Line of Best Fit / Slope:** Thinking that a line is a good estimate for data that is not linear; Drawing a line that is not a good fit for the data, and calculating the equation of this line; Miscalculating the slope of a line using two points on the line; Incorrectly calculating the y-intercept when finding the equation of a line given a graph; Using an exponential function to estimate a relationship between two variables when a linear function is a better fit; Confusing x and y when graphing data points or analyzing a graph.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: Summarize categorical data for two categories in two-way frequency tables; interpret joint relative frequencies in the context of the data; fit a linear function for a scatter plot that suggests a linear association

Accelerated: Interpret conditional relative frequencies in the context of the data; find and use linear models to solve problems in the context of the data

Advanced: Use functions suggested by the data to solve problems in the context of the data

Ohio's Learning Standards – Clear Learning Targets
Algebra 1

S.ID.7, S.ID.8, S.ID.9

Interpret linear models.

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of data.

Compare (using technology) and interpret the correlation coefficient of a linear fit.

Distinguish between correlation and causation.

Essential Understanding

Students should be able to address three questions related to correlation vs. cause (causation) when analyzing bivariate data: Is there a correlation between the two sets of data? Is the correlation weak or strong? Is the correlation positive or negative?

Students should understand correlation as the degree of fit between two variables.

Students should be able to create a mathematical mode of a situation, test and improve the model, clearly communicate their reasoning; and evaluate alternative model of the situation.

Extended Understanding

Introduce technology in this section for ease of obtaining solutions or visualizing results. Many stats packages are available that will perform regression with multiple variables.

Vocabulary

- Association
- Bivariate data
- Box plot
- Categorical data
- Causation
- Center
- Correlation coefficient
- Dot plot
- Frequency table
- Histogram
- Interquartile range
- Joint frequency
- Line of best fit
- Outlier
- Qualitative (numerical) variable
- Quantitative (numerical) variable
- Regression
- Residual
- Scatterplot
- Spread
- Standard deviation
- Symmetric
- Two-way frequency table

Essential Skills

- I can explain the meaning of the slope and y-intercept in context
- I can use a calculator or computer to find the correlation coefficient for a linear association; I can interpret the meaning of the value in the context of the data; I can explain the difference between correlation and causation.

Instructional Strategies

Some students may have difficulty in clearly distinguishing between correlation and causation. Provide multiple opportunities for students to see examples of each. Provide opportunities for students to formulate meaningful questions in the first step of the four-step process. This process takes time and lots of practice and guidance.

Additionally, students need practice creating these questions as it relates to real-world contexts.

To make some sense of Pearson's r, correlation coefficient, students should recall their middle school experience with the Quadrant Count Ratio (QCR) as a measure of relationship between two quantitative variables. Noting that a correlated relationship between two quantitative variables is not causal (unless the variables are in an experiment) is a very important topic and a substantial amount of time should be spent on it.

Common Misconceptions and Challenges

A fallacy that is often committed in analyzing and interpreting data is to errantly draw conclusions about cause and effect once a correlation is found. A strong correlation simply tells whether a relationship exists between two sets of data. It does not indicate why the two sets of data are related or if changing one of the sets of data will cause the other set of data to change

Residuals: Incorrectly finding the residual; Incorrectly plotting points on the residual plot. **Causation:** If a strong correlation indicates that one event causes another; Failing to consider outside factors that may influence the strength of the correlation between two events. **Correlation**: Using the correlation coefficient to analyze data that in not linear; Incorrectly using the correlation coefficient to assess the strength of a relationship.

Performance Level Descriptors

Limited: N/A

Basic: N/A

Proficient: Distinguish between correlation and causation; compute (using technology) the correlation coefficient of a linear fit

Accelerated: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data

Advanced: N/A